BLESSED JORDAN'S CONTRIBUTION TO MATHEMATICS

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WITH the dawn of the present century there was already at work upon the trend of historical research a new influence, an influence which has inspired latter-day historians to rise as far as is humanly possible above personal prejudice, acquired or inherited; to transcend the narrowing limits of nationalism, religion and race. For four centuries immediately preceding, that is to say, from the time of the Reformation, history had been, in the words of Comte de Maistre, "one great conspiracy against the truth." Especially was this true in all that regards the Catholic Church and her attitude towards science. Only the critical viewpoint of modern history, insistent on examining original manuscripts, has convinced present day students that the discoveries of Church scholars form in large part the bed-rock foundation on which is built the edifice of modern science.

Looking about for a reason for all the obscurity, and in many cases out-and-out deception, we are forced to conclude that it was mainly the work of the "Reformers." For with the rise of the Reformation there also arose a new body of Protestant tradition in direct contrast to that of the Church. These Reformers were entirely occupied with an attempt to place their religious rebellion in a favorable light in the eyes of the educated world. To accomplish this purpose they spared no effort, going to unbelievable extremes to blacken the most luminous pages of Catholic achievement. Subsequent generations, however, were more often deceived than deceiving.[1] Already deeply entrenched in a prejudicial tradition, they had no reason to doubt the learned men to whom they looked for leadership.

But all of this confusion and much of the detrimental prejudice is being gradually battered down by a steady barrage of truth and original research. What has already been done merits high commendation and leaves one somewhat hopeful of a com-

[1] James J. Walsh, *The Church and Science*, p. 24.

pletely fair-minded judgment of the Church's part in the development of science in the not too distant future.

Modern historians, working with this new, broad and unbiased perspective, have given to many of the Church's most accomplished sons a merited, though belated, recognition. This is more especially true of those who labored in the field of science. Not the least among these is the Dominican, Blessed Jordan of Saxony, or Jordanus Nemorarius as he is sometimes called. While the purpose of the present paper is principally to treat of Jordan as a mathematician, it might be well first to review briefly his life. He was born in Borgentreich, in the diocese of Paderborn, a mountainous and woody country from which he received the nickname *Nemorarius*, which freely translated means yokel.[2] Père Berthier, O. P., places the date of his birth at about 1190. We find him in 1220 at the University of Paris receiving the habit of a Friar Preacher from Father Matthew of France. Already he had a reputation for learning and had distinguished himself in philosophy, mathematics, literature, scripture and theology.[3] The following year, 1221, the general chapter of the Order at Bologna appointed him provincial of Lombardy. So marked was his ability and sanctity, and so great was the confidence in which he was held by the brethren, that in 1222, upon the death of St. Dominic, he was elected to be the second Master General of the Order.

Père Mortier has given us a very capable account of his personal character. "God blessed him generously," he tells us, "with those masterful qualities which enchant and captivate others. Jordan was a charmer of men. He possessed those bed-rock virtues which compel respect and enforce confidence—personal austerity, angelic purity of heart, nobility of soul, an unswerving spirit of justice, heroic forgetfulness of self. Providence enriched a strong mind with the most lovable attractions. His speech was ready and gracious. It scintillated with wit, and was as sharp as a sword. In case of need it could strike the terror of a clap of thunder. This gift he used with the greatest skill, going straight to the point which he wished to inculcate. Affable and easy of approach, kindly in his ways, ever in good humor (often even jovial), his gentle nature disarmed all wrath. He was an ideal Friar Preacher—a true type of the Order."[4]

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[2] Moritz Cantor, *Vorlesungen uber Geschichte der Mathematik* (Leipzig, 1892) II Band, Kap. 43, p. 57.

[3] Quetif-Echard, *Scriptores O.P.*, p. 98;
V. F. O'Daniel, O.P., *The First Disciples of St. Dominic*, p. 400.

[4] Mortier, *Histoire des Maitres Generaux*, I, 143.

His chief delight was to work among the young. His letters to Blessed Diana reveal that he seldom failed to receive from twenty to thirty novices into the Order around the Easter time, the fruit of his alternate courses of Lenten sermons at Paris and Bologna. Some idea of the incredible industry of the man, and of his dynamic energy, may be had when we realize that, during his term as Master General of the Order, he founded four new provinces, established sixty new convents throughout Europe and the Holy Land, personally clothed over one thousand novices with the habit, and did all this in the face of the most formidable opposition.

Germany was ruled by the stubborn and perverse Frederick II who was patronizing forbidden Jewish and Saracen philosophy, fostering the astrological speculations of Michael Scot and keeping a Saracen harem at court, chiefly because it was singularly offensive to the Pope with whom he was then engaged in a quarrel. It is significant that Dante, who was almost his contemporary and who obtained his knowledge of him from those who knew him personally, places him in the Inferno with Epicurus and his followers. Yet Blessed Jordan had the courage to berate personally, this man who had for so long struck terror into the hearts of his subjects. He called him to task for his shameful conduct toward the Pope, Gregory IX, for his superstitious belief in auguries, for his patronage of the Jews and Saracens and persecution of the Christians. He concluded by exhorting him to a complete change of conduct and to a serious concern for the salvation of his immortal soul. Frederic II, admiring the man's fearlessness, was converted (though his contemporaries doubt it), and on his death bed was clothed in the Cistercian habit.[5]

Over France ruled Philip Augustus, cold, unscrupulous, a constant menace to the program of Innocent III for reform. The Holy Father was forced to lay the country under interdict until Philip repudiated his unholy relations with Agnes of Meran and took back his lawful wife, Ingeborg of Denmark. In Spain, Alfonso also had to be whipped into line and forced to break off the marriage contract he had entered into with his own niece. And in England the notorious and recalcitrant John was dead only nine years.

[5] *Acta Sanctorum,* V, 733, no. 52; Fleury, XVII, 144.

Such were conditions in the Europe where Jordan labored so successfully, a Europe steeped in a complete moral indifference, especially on the part of its chief rulers. From such men Blessed Jordan and his Friars could expect little encouragement and no practical help in their enormous task of regenerating the tenets of Christian morality throughout the western world. Their difficulty did not stop here however. For over and above this general decline of morality they were confronted by the voracious monster of the Manichean heresy which sought to devour what vestiges of Christian life the menace of indifference had left. The added momentum received from returning apostate crusaders and the entrance into the West effected through the rising commerce of Venice helped to spread this heresy over the entire face of Europe.[6] Its devotees covertly propagated their religious obscenities and held midnight assemblies at which the most disgraceful debaucheries were consistently practised. Subsequently they divided into some seventy-six sects of which the best known were the Albigenses, the Cathari or Puritani, the Luciferians, the Paulicians and the Paterenes, with whom at a later period a large number of the Waldenses became partially identified.[7]

One is amazed at the far reaching influence of this gifted and hard working Dominican Friar, who labored in an atmosphere of such universal antagonism. He converted hundreds to a better moral life. He dispelled heretical doctrines from the minds of the most learned men at the universities of Paris, Bologna and Oxford. We have already seen the fruitful results of his labors for his own beloved Order. The marvel of it all is that, despite such unceasing labor, wearying journeys on foot to every part of Europe, days of prayer and preaching, despite all of this, he found time to withdraw to the quiet of his cloister and there discover mathematical principles which were epoch making in the development of that difficult science.

His accomplishments in the field of science have been more or less overshadowed in history by his preeminence as an organizer, as a preacher, and as a theologian. An English chronicler of the fourteenth century, Nicholas Trivet, under the year 1222, tells us that even so early in his career Jordan was considered

[6] Fletcher's notes on 1st letter of DeMaistre on the Inquisition, Mosheim, *Eccl. Hist.* V, III.

[7] Everini, Epist. in Bernardi Opp. I, 1492; (Dollinger).

great in the profane sciences and especially so in mathematics.[8] But before proceeding to a consideration of the actual works of Blessed Jordan it might be interesting to examine the chain connecting the mathematics of the classical era and of the thirteenth century.

The first name of prominence we find at the end of the fifth century, that of Boethius (475-526), whose mathematical works might better be called compilations. They comprise a geometry consisting of excerpts from the first, third and fourth books of Euclid,[9] and a number of treatises on arithmetic founded on the work of Nichomachus the Jew. Cassiodorus (480-566) contributed a quadrivium of arithmetic, geometry, music and astronomy. It has always seemed quite paradoxical to the writer and not a little amusing, that music, "that house in which is held communion with the infinite," that medium of expression which has conveyed to men all the warmth of inspired beauty and emotion, the delicate tone pictures conceived in the souls of artists, is to the scientist but a subdivision of the hard, cold science of mathematics.

St. Isidorus (d. 636), who became bishop of Seville, also devoted the third volume of his encyclopedic twenty volume *Origines* to the mathematical quadrivium. He gives definitions and grammatical explanations of technical terms but no description of the modes of computation then in use. The subsequent century of darkness was finally dissipated by the appearance of Bede the Venerable (672-735), a native of Wearmouth, England, and the most learned man of his time. He is responsible for several erudite tracts on *Computus*, or the computation of Easter time, and on finger reckoning. It appears that a finger system was then widely used for calculation. Bede was followed by Alcuin (735-804), an Irish monk, who because of his renown as a mathematician was called to the court of Charlemagne.[10]

In the confusion attendant upon the disintegration of Charlemagne's mighty empire, scientific pursuit was abandoned until the tenth century. Gerbert the Aquitanian (950-1003), who be-

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[8] Nicholas Trivet: "Hoc anno in Capitulo Fratrum Praedicatorum Generali tertio, quod Parisiis celebratum est, successor Beati Dominici in Magisterio Ordinis Fratrum Praedicatorum factus est Frater Jordanus, natione Teutonicus, Dioecesis Moguntinae, qui cum Parisiis in scientiis saecularibus et praecipue in mathematicis magnus haberetur. . . ."

[9] W. W. Rouse Ball, *History of Mathematics*, "Boethius."

[10] Florian Cajori, *History of Mathematics*, "Mathematics of the Middle Ages."

came bishop of Rheims and later Pope as Sylvester II, was chiefly responsible for the renewed zeal of the monks in their study of mathematics during this period. He discovered the geometry of Boethius and made it the text book for all Europe. His writings consist of a treatise on arithmetic, another on geometry, and still another on the use of the abacus. The abacus, as described by Berenelius, a disciple of Gerbert, was a smooth board on which geometricians were accustomed to strew blue sand in which they traced their diagrams.[11] However the abacus as generally used consisted of a wooden frame across which were suspended beads or counters much in the same manner as those at the top of children's slates as an aid in learning to count. The rules for its use were most intricate. Gerbert's work on geometry is said to be of unequal merit, but in the course of his discussion he corrects several errors made by Boethius and solves several propositions of remarkable difficulty for the time. One of these problems is to find the sides of a right angle triangle whose area and hypotenuse are given.[12] His rules for division are the oldest extant.

From this brief retrospect we learn that, with the single exception of Gerbert, no writer added anything of importance to the sum of mathematical knowledge from the classical period to the beginning of the twelfth century. It may be said in full truth that even at the close of that century mathematical learning was of a most elementary and fragmentary character.[13] But then was born a new and more intellectual era in the progress of science, which gave us mathematicians of the stamp of Blessed Jordan and Leonardo of Pisa, the Venetian merchant; a jurist in Raymond of Pennafort, whose work in canon law was the mainstay of church law till as late as 1918; and in Jordan's illustrious pupil, Albertus Magnus, the most comprehensive scholar of the middle ages, who merited for himself the title *Doctor Universalis.*

Up until the close of the last century Blessed Jordan's mathematical works were almost entirely unknown.[14] However his star is once again in the ascendency, mainly through the exhaustive research of Professor Maximillian Curtze of Thorn, and of Mortiz Cantor whose monumental work of four volumes, *Vorlesungen uber Geschichte der Mathematik,* seems to be the prin-

[11] *ibid.*

[12] W. B. Cahill, "A Famous Medieval Mathematician," *The Rosary Magazine,* Dec. 1905.

[13] *ibid.*

[14] W. W. Rouse Ball, *op. cit.,* p. 177.

cipal source book for all modern history of mathematics. From their elaborate investigations we learn the interesting fact that we owe to Jordan of Saxony the invention of syncopated algebra in which letters are used for algebraical symbols. Hints of this practise were already made in the works of earlier writers including Aristotle, Diophantus and, in one instance, Leonardo of Pisa. But Jordan used letters quite as they are used to-day, letting "b", for example, represent any number whatsoever.[15] He also made use of letters to demonstrate the rules of arithmetic as well as algebra. In this practise the learned Dominican was far in advance of his contemporaries.[16]

His work *De Numeris Datis*[17] is a system of algebraic rules and one of the leading treatises of the Middle Ages on algebra.[18] It is divided into four books and contains solutions to one hundred and fifteen problems. The problems generally relate to a *numerus datus* (a number whose quantity is known) which has to be divided in some stated manner, as in many of the problems of the present algebra text books.[19] One of his first problems consists practically in the division of a given number into two parts so that the sum of the squares of the parts shall be another given number.[20] Others of these problems lead to simple or quadratic equations involving more than one unknown quantity. He shows a knowledge of proportion surprising for that era.

In geometry he is represented by three works, *De Triangulis, De Similibus Arcubis,*[21] and *De Isoperimetris,*[22] of which the first named is the most important. This is divided into four books. In the first, besides a few definitions there are thirteen propositions on triangles based on Euclid's "Elements." The second contains nineteen propositions mainly on the ratios of straight lines and the comparison of the areas of triangles; for example, one problem is to find a point inside a triangle so that the line joining it to the angular points may divide the triangle into three equal parts. The third book contains twelve propositions chiefly concerning arcs and chords of circles, while the last book has twenty-eight propositions, partly on regular polygons and partly

[15] Cantor, *op. cit.*, II, (2), 56; G. Enestrom, *Bibl. Math.*, VII, (3), 85.
[16] W. W. Rouse Ball, *op. cit.*, p. 177.
[17] Published by P. Teutlein in 1879 and edited in 1891 with comments by Max. Curtze in Vol. XXXVI of the *Zeitschrift fur Mathematik und Physik.*
[18] W. W. Rouse Ball, *op. cit.*, p. 178.
[19] P. Treutlein, *Abhandlungen* II, 135, (1879).
[20] *ibid.*, p. 136.
[21] Published by Curtze of Thorn in Vol. VI, *Mitteilungen des Copernicus-Vereins zu Thorn,* (1887).
[22] Published by Curtze, Vol. XXXVI, *Zeitschrift fur Mathematik,* (1891).

on miscellaneous questions such as duplication and trisection problems.

The *Algorithmus Demonstratus*[23] of Blessed Jordan contains practical rules for the four fundamental processes. By a general use of Arabic numbers in this treatise he gives evidence of the progressive mind so characteristic of all really great men, in readily accepting the worthwhile innovations and discoveries of others. (This system of Arabic numerals had only shortly before been introduced into general use by Leonardo of Pisa.) He divides this work into ten books dealing with the properties of numbers, primes, perfect numbers, etc., ratios, powers, and progressions. It would seem from it that Jordan knew the general expression for the square of any algebraic multinomial.[24] Until the last part of the eighteenth century this work was universally attributed to Regiomontanus (1436-1476). Now once again its authenticity has been denied by G. Enestrom[25] but such an eminent authority as David E. Smith, writing in 1923, has not seen fit to accept this statement unrestrictedly.[26] However even though it should be proved to be not the work of Jordan, it would derogate in no way from his reputation as a mathematician, nor from his just renown as the discoverer of syncopated algebra which he used chiefly in his works *De Numeris Datis* and *Arithmetica decem Libris Demonstrata*.[27]

Jordan's knowledge was by no means limited to arithmetic, geometry and algebra, for he was also well versed in the astronomy, mechanics and optics of his age. The *Tractus de Sphera* was for a long time a classic and went through several editions.[28] His efforts in mechanics are embodied in his work entitled *De Ponderibus Propositionibus XIII*, which was printed at Nurnberg in 1533 and contains among other things a brief treatment of statics.

Many authors have used Jordan's works without due acknowledgement; some in sheer plagiarism, others because of historical misinformation. One source of this lack of recognition is had in the fact that a Franciscan Friar, Lucas de Paciola of Venice (d. 1510), wrote a *Summa de Arithmetica Geometria Proporzioni e Proporzionalita* which was the first book printed on arithmetic and algebra in Venice (1494). It thus antedated the

[23] Published by Curtze in Vol. XXXVI, *Zeitschrift fur Mathematik*, 1891.
[24] W. W. Rouse Ball, *op. cit.*, p. 178.
[25] G. Enestrom, *Bibl. Math.*, V, (3), 9.
[26] D. E. Smith, *History of Mathematics*, (Boston, 1923), p. 226.
[27] Published by Faber Stapulensis, at Paris, 1496; second edition, 1514.
[28] D. E. Smith, *op. cit.*, p. 227.

printing of Jordan's *Algorithmus Demonstratus* (1534) by a full forty years and so gave rise to a proneness on the part of scholars to credit Paciola with the abler work of the medieval Dominican.[29] Paciola used abbreviations but did not rise to Jordan's conception of representing known quantities by letters as is done also in modern algebra.

Probably the most interesting of such tacit exploiters of Jordan's genius is one Michael Stifel, an Augustinian who was beguiled by Luther's eloquence into apostatizing from the faith. Overzealously applying his pet theory of numbers to biblical interpretation, he had, by a series of comical conclusions, thoroughly convinced himself that Pope Leo X was the beast mentioned in the Apocalypse. His numerical interpretations came to grief however when his parishioners in the village of Halzdorf took his word that the world was to end on October 3, 1533. Some of them gave themselves up to prayer, others dissipated, and all abandoned their work. When the day foretold had passed, the peasants, furious at the deception, seized the unfortunate prophet and he was saved from violence at their hands only by being arrested and thrown into jail at Wittenberg. Luther later used his influence to get him out.

Stifel's chief work is an *Arithmetica Integra,* published in Nuremberg in 1544, to which Melancthon wrote the preface. He devotes the third book to algebra and this little book has proved his undoing, for in it he uses without the slightest acknowledgement the work of Blessed Jordan, transcribing him almost verbatim.[30] His discussion therein of known and unknown algebraic quantities, and his use of *a, b, c,* etc., to represent the unknown, bear so striking an analogy to Jordan's treatment of the same subject that modern criticism denies him any further consideration. However, in his own underhanded way Stifel, as one author so mildly puts it, did serve to "reintroduce the general algebraic notation which had fallen into disuse since the time of Jordan."[31]

Just how much influence the mathematical genius of Jordan of Saxony has exercised upon the development of algebra and geometry is disputed. That it has been not a little is evident from the work of Moritz Cantor who has gathered a most complete record of references and evidences of such an influence. An exhaustive enumeration of them here would be too lengthy and quite uninteresting.

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[29] W. W. Rouse Ball, *op. cit.,* p. 217.
[30] Cantor, *op. cit., Kapitel* LXII.
[31] W. W. Rouse Ball, *op. cit.,* pp. 221-222.

The adverse opinion, namely that his influence was practically insignificant, is founded upon the fact that notwithstanding the availability of his works to students for over two hundred years, comparatively few derived much benefit from them and little was done to extend the bounds of arithmetic and algebra as there set forth. One explanation of this may be found in the Church's disapproval of Arabian philosophers and scientists who were openly antagonistic to the faith. This should not be construed as a rejection of science. For it was not science or philosophy that the Church frowned on but rather Arabian translations of the original Greek masters. In their translations the Arabs did not adhere closely to the original texts, but interpreted and read into them their own applications of the authors' principles. When however the original copies of Aristotle and others were found and mastered this ecclesiastical caution was withdrawn.

It must also be remembered that for the two centuries immediately subsequent to Jordan's time philosophical and theological studies rather than mathematical and scientific pursuits preempted the field of higher educational investigation. Nor must it be forgotten that during those two hundred years of indifference to his mathematical achievements no mathematician of the same ability as Blessed Jordan appeared to take up the work as he had left it. Jordan stood quite alone. His accomplishments do not imply the general standard of mathematical knowledge then current. After his death in 1237[32] his invaluable discoveries fell into disuse for the very reason that they were not understood. His manuscripts were scattered and his fame was obscured by the rising lights of scholasticism.

Time, though tardy, is now doing justice to the genius and the distinguished work of Blessed Jordan, even though names once honored in the history of science, both historical and mathematical, have had to suffer in the process. Modern historians, and among them many non-Catholics, have worked with a zeal and fair-mindedness that has transcended personal prejudice to a most satisfying degree. They have elevated Jordan of Saxony to a place beside his more famous pupil, Albertus Magnus, and the distinguished Franciscan, Roger Bacon. With them he stands as a monumental refutation of the diminishing though persistent bugaboo of the Church's opposition to modern science.

[32] Blessed Jordan with two companions was drowned in a shipwreck while returning to Europe from his visitation of the province he had established in the Holy Land. (V. F. O'Daniel, *op. cit.*, p. 429.)